When will we see people of negative height?

The heights of human beings follow the normal distribution. Or do they? If they do, say **Patrik Perlman** and colleagues, why don't we see human beings with negative heights?

Every elementary textbook has it: the heights of men and women follow the normal distribution. Lots of people centre round a mean value, currently 1.776 m (5 ft 10 in) for grown American men, 1.632 m (5 ft 4½ in) for grown American women[1]; on either side of that value the number of people tails off. A minority of men are 1.90 m (6 ft 3 in) tall, fewer still are 2.00 m (6 ft 6½ in), very few indeed are 2.20 m (7 ft 2½ in) tall – and the number who are 2.40 m (7 ft 10½ in) feet tall is absolutely tiny. And the same is true on the shorter side: not so many adult men or women are less than 1.50 m (4 ft 11 in), fewer still less than 1.30 m (4 ft 3 in).

Graph it out and you get a bell curve, a normal or Gaussian distribution, the standard shape so ubiquitous in nature, so beloved of statisticians. And, it would seem, such a natural one to apply to human beings.

But is the normal distribution really the correct one for describing and analysing the various heights of human beings? The Statistics Department of the University of Dortmund was founded in 1973, and in its early days this topic was much discussed – and with passion. Those in favour of the normal distribution had much on their side: it fits the observed data beautifully; it is useful, and usable, and elegant. But as those against point out, it has one strange and unnerving consequence: it predicts that there should be human beings of *negative height*.

The argument is obvious, and clear. Figure 1 shows a normal distribution, centred around a mean height. The right-hand tail shows the decreasing numbers of people whose heights are tall, then very tall, then extraordinarily tall; the left-hand tail tells the similar story for shorter people. And the people in the area shaded blue are not just very short: their height is less than zero. They are men and women of negative height.

Do these people exist? Can we see them? Have they ever been observed? And if they do exist, what of their other attributes? Is their weight – strictly, their mass – negative as well? Negative mass is certainly a physical possibility. The concept in science goes back to the eighteenth century, and negative mass is still subject to research activities in recent years. Yet, curiously, negative height – and especially negative body height – seems not yet to have been considered in common scientific journals. If it is related to negative mass this raises questions which should be of great interest to physicists, as well as to the friends and relations of the human beings so afflicted. As just one example, the cosmological theory of wormholes in space-time is closely related to the existence of negative mass as wormholes need this kind of mass in order to become stable and stay open. In this context, it is surprising that negative height did not stimulate minds in the same way. Would a human with negative body height have negative body mass? If so, would he be able to create wormholes offhand and travel the universe freely? Do these creatures already exist? Have they left us behind and gone off to a planet, less polluted and with a climate like the south of France? Is that why we have never found them?

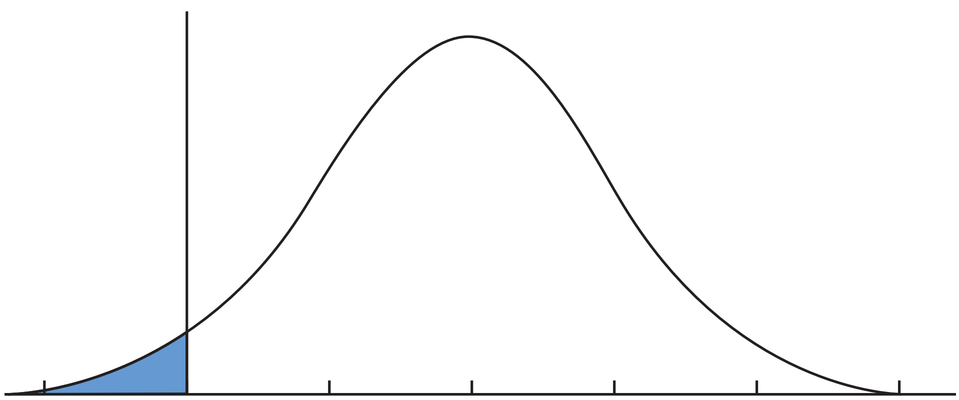
But what is the relation to the discussion about the normal distribution? As the famous statistician George Box said, "All models are wrong, but some are useful." One argument against using the normal distribution as a model for human heights relies on its inappropriateness due to the possibility of negative values. But our question is, do we really have enough evidence to exclude the occurrence of a negative body height?

In this article, body height will be understood as body height of fully grown adults. Human growth curves generally show changes of body height until the age of around 20 years. Consequently, this age will be considered as the cut-off age for the fully grown population. (The question of negative height among children is problematical: in which direction would the child grow?)

Our question is then: do we have enough evidence to exclude the occurrence of negative body height among adults?

Let us look again at the assumptions of our normal distribution. Certainly any human being of negative height would be a large number of standard deviations away from the mean height. This makes them rare. So how many standard deviations away from the mean would a person of negative height be? And how rare would that make them? The answers are easy to calculate when the mean and standard deviation are known.

First of all, what is our data? We should consider as large a database as possible – in other words, all the people in the world. Human heights, and the extent to which those heights vary, clearly depend on the person's gender and



A normal distribution of human heights. Heights increase to the right. The mean is around 1.77 m; decreasing numbers of people are very tall, or very short. The blue shaded area represents people of negative height also vary largely between different countries and populations[1]. The mean height of Mbenga tribesmen of Central Africa is under 150 cm (4 ft 11 in); for Polynesians of Samoa and Tonga it is 180 cm (5 ft 11 in). We can get round this complication by taking as our measure of variation not the standard deviation but the coefficient of variation (CV), which is the standard deviation divided by the mean. If the CV is roughly the same between men and women and between countries, it would help us considerably in our sums. It is not an unreasonable assumption, and we shall make it. So what is the value of that coefficient?

US data[1] suggest that the CV for 18 688 women and 17 316 men is 5.4% and 5.2%, respectively. Another international database of 20 regions in the world shows CVs of 5.2% for women and 4.4% for men. Based on these data, we shall take 5.4% as an upper boundary for our CV. In other words, the standard deviation of a population's height is 5.4% of its average height. For American males, whose average height is 177.8 cm (5 ft 10 in), this means the standard deviation in their height is 9.6 cm or 3.78 in.

That gets us a bit further in our quest for the person of negative size. For our American population – and for the world population in general – a person of zero height is 5 ft 10 in – that is, 18.5 standard deviations - below the mean.

This makes them rare. How rare? Again the answer is not complicated. The normal distribution is, as we have said, the most studied in history. Tables have been calculated to tell us exactly that. Table 1 gives us a guide.

In a normal distribution 68.26% of the population lie within 1 SD of the mean, so 31.74% are more than 1 SD from the mean; 95.44% lie within 2 SDs of the mean, so 4.56% are more than 2 SDs from the mean; and 99.73% lie within 3 SDs of the mean, so 0.27% are more than 3 SDs from the mean.

So what proportion is more than 18.5 SDs away from the mean? Standard software will tell us the answer: 1.03×10^{-76}. It is a small figure, but still a positive one. That is, on average one person in 9.71×10^{75} will fit our criteria. We can without much loss of accuracy round that to one person in 10^{76}.

Table 1. Proportion of the population within 1, 2 or 3 standard deviations from the mean in a normal distribution

Number of standard deviations from the mean	*Percentage of the population within that range*	*Percentage outside that range*
1	68.26%	31.74%
2	95.44%	4.56%
3	99.73%	0.27%

This is the probability of a person being 18.5 SDs below the mean (i.e. of zero or negative height). It is also the probability of a person being 18.5 SDs *above* the mean – which would put him or her at more than 3.55 m, or 11 ft 8 in, tall. Literature – children's literature at least[2] – frequently tells of giants of such stature, but elementary physics tells us that their leg bones would be too weak to support their great weight and that they would collapse in a heap crushed by their own weight. So we shall reject the idea of anyone of that height as ridiculous.

So our hypothesis is that if the normal distribution is the correct one for human heights, we may expect on average to find one person in 10^{76} who is of zero or negative height. This raises three further obvious questions:

1. Why have we not seen such a person yet? Or, to put it another way, what is the probability of observing a fully grown adult with negative body height among people (a) living today, or (b) who have ever lived on earth?
2. How many people are necessary to have ever lived on earth in order for there to be at least a 95% probability that one of them has been of negative height?
3. By when can we expect to reach the necessary number of people who ever lived on earth from question 2?

We answer these questions below.

What is the probability of there being a fully grown adult with negative body height among the people living today?

The world population crossed the barrier of 7 billion people living on earth on 31st October 2011[3]. The *World Factbook* of the Central Intelligence Agency[4] reports an estimate that the percentage of people aged 15 years or older is 73.7%. This is not quite the age cut-off that we defined above for the fully grown population, but we can use it to give an upper boundary for today's adult population, which we can therefore estimate to be 7.02 billion × 0.737, which is about 5.17 billion adults.

Standard software packages can usually calculate the sort of probability we need, that of finding our one person in 10^{76} when we have a population of 5.17 billion in which to look for them. Unfortunately, standard software packages meet their limits here. Because of the extremely tiny probabilities involved, they round them down and return an answer of zero.

However mathematical techniques can get us round this. (Technically, we use log transforms and Taylor expansions. Full details can be found on the *Significance* website.) The answer turns out to be that the probability of one person of negative height existing among the 5 billion currently living on earth is 5.33×10^{-67} – or one chance in 2×10^{66}. To our knowledge, nobody has yet reported seeing such a person. Looking at this tiny probability, this is hardly surprising. It is certainly not evidence enough to reject our hypothesis that one person in 10^{76} is of negative height.

What is the probability of there having been a fully grown adult with negative height among the people who have ever lived on earth?

It has been estimated[5] that the number of people who have ever lived on earth is 107.6 billion. However, we cannot know what proportion lived long enough to grow to adulthood and their full height. Therefore, we must take this number of 107.6 billion as an upper boundary.

Using the same techniques as for the previous question yields an upper estimate of the probability that at least one fully grown person with negative height has ever existed. It is 1.11×10^{-65}.

To our knowledge, no person with negative body height has been reported from the beginning of mankind, but this is hardly surprising, given its tiny probability. It is still not evidence to reject our hypothesis that one person in 10^{76} is of negative height. However, this has to be interpreted with care as we cannot be sure that a case (or even cases) of negative body height in the past, and especially in the early times of humankind, would have been recorded and would have come to our knowledge. It might even be that negative body height was nothing rare in the very early days of mankind but that positive body height was an evolutionary advantage and therefore this characteristic became extinct. In 2003 remains of a tiny, 3-foot-high race of humans nicknamed "hobbit-men" were found on the island of Flores in Indonesia (*Significance*, December 2009). Archaeologists may yet unearth a hominid skeleton of negative height.

How many people are necessary to have ever lived on earth in order to observe at least one fully grown adult with negative body height with a probability of at least 95%?

Let us now take a look into the future. It is easier for software to answer the question of how many people will have to have been born before we can expect with 95% certainty that one of them will have negative body height, because the number is not tiny. In fact it is huge. It turns out to be 2.9×10^{76}, which in words is 29 thousand billion billion billion billion billion billion billion billion people.

Writing that number is easy; comprehending it is much harder. For example, a computer of the current generation, able to perform 10.5 petaflops (10.5×10^{15} floating-point operations per second), would need 8.75×10^{52} years just to count to this number. This is much longer than the age of the earth, which is estimated to be around 4.6 billion years. The Big Bang is estimated to have occurred about 13.7 billion years ago.

By when can we expect to reach this necessary number of people ever lived on earth?

When, in the continuing (we hope) history of mankind, this vast number of human beings shall have been born and reached adulthood, we shall at last have sufficient evidence to decide upon our hypothesis that human heights follow a normal distribution. If at that point no one of negative height has been observed, it will be evidence enough to reject the idea. So how long do we have to wait for this number to have ever lived on earth? For this we need to estimate the future evolution of the human population.

In 2004 the United Nations[6] tried to predict the world population. However, their estimates do not go beyond the year 2300. We need a much larger time horizon. Therefore, we must rely on the extremely simplified (and almost certainly unjustified) assumptions of a constant exponential growth based on the current global birth rate of 19.14 births per 1000 per year and the current global death rate of 7.99 deaths per 1000 per year[4]. This gives us an assumed constant growth rate of 11.15 per 1000 per year, which is 1.115%.

On this assumption, and starting from a population of 7.02 billion people living in July 2012[4], an estimate for the number of people alive at time t (years since 2012) is given by $N(t) = 7.02 \times 10^9 \times 1.011\,15^t$.

To get from this to the number of people who have ever lived on earth we have to take into account that an individual contributes in each year of his lifespan to the number of people alive. We have to divide $N(t)$ by the mean future life expectancy at birth.

The current global life expectancy is reported[4] to be 67.59 years. However, this number cannot be used as an estimate for the future. Life expectancy differs greatly between countries (mainly influenced by differences in medical care) and shows a large change over time[4]. Global life expectancy has increased by around 20 years since 1950[7]. Therefore, it can be expected that life expectancy in less developed countries will continue to increase to

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attain the level of the developed countries in the near future. But the developed countries have also shown an increase in life expectancy over the last 60 years[7], from 65.9 years to 76.9 years. In order to account for this future development we will use an estimate of 100 years for the future global life expectancy at birth.

Given this estimate, we can work out how long we must wait until 29 thousand billion billion billion billion billion billion billion billion people have ever lived. Again the mathematics is on the website for those who want it. It turns out that the last of those people will be born and attain adulthood around 13 842 years from now, in the year AD 15855.

Discussion

We have shown that the normal distribution, combined with current information available on height of fully grown adults, is just not enough to exclude the possibility that a person with negative body height could exist. We must wait until the year AD 15855 before we will have enough evidence to judge whether we must abandon the normal distribution hypothesis, or else accept the existence of at least one negative height human being. It is good news in a way. We have only 13 842 years to wait before we have a sufficiently large database to decide.

Alternatively, if we cannot wait that long, we could always extend George Box's quote: All models are wrong, but some are useful – for part of their range at least.

Acknowledgements

The authors Patrik Perlman, Beat H. Possen, Veronika D. Legat, Arnd S. Rubenacker, Ursel Bockiger and Lisa Stieben-Emmerling wish to thank their statistics teachers at the University of Dortmund (in randomised order): Siegfried Schach, Joachim Kunert, Iris Pigeot-Kübler, Joachim Hartung, Wolfgang Urfer, Franz Hering, Claus Weihs, Ursula Gather, Andreas Christmann, Sigrid Pöhlmann.

References

1. McDowell, M. A., Fryar, C. D., Ogden, C. L. and Flegal, K. M. (2008) Anthropometric Reference Data for Children and Adults: United States, 2003–2006, National Health Statistics Reports, No. 10, October 22nd. Hyattsville, MD: Centers for Disease Control and Prevention, National Center for Health Statistics, US Department of Health and Human Services. Available at `http://www.cdc.gov/nchs/data/nhsr/nhsr010.pdf`.
2. Tabart, B. (1807) *The History of Jack and the Bean-stalk*. London: Juvenile and School Library.
3. United Nations Population Fund (2012) *State of World Population 2011 – People and Possibilities in a World of 7 Billion*. New York: United Nations Population Fund. Available at `http://foweb.unfpa.org/SWP2011/reports/EN-SWOP2011-FINAL.pdf`.
4. Central Intelligence Agency (2012) *The World Factbook*. Washington, DC. Available at `https://www.cia.gov/library/publications/the-world-factbook/index.html`.
5. Haub, C. (2011) How many people have ever lived on earth? Population Reference Bureau< Washington, DC. Available at `http://www.prb.org/Articles/2002/HowManyPeopleHaveEverLivedonEarth.aspx`.
6. United Nations (2004) *World Population to 2300*. New York: United Nations, Department of Economic and Social Affairs. Available at `http://www.un.org/esa/population/publications/longrange2/WorldPop2300final.pdf`.
7. United Nations (2011) *World Population Prospects, the 2010 Revision* (updated 28 June 2011). New York: United Nations, Department of Economic and Social Affairs. Available at `http://esa.un.org/unpd/wpp/Excel-Data/mortality.htm`.

The authors studied statistics at the University of Dortmund. The lead author is 181 cm tall.